CHAPTER 2

Using Cognitively Complex Tasks in the Classroom

A major aim of the QUASAR Project was to provide students with increased opportunities for thinking, reasoning, problem solving, and mathematical communication. Student learning could not be expected to deepen or become more conceptually rich, it was argued, unless students were regularly, actively, and productively engaged with cognitively challenging mathematics.

Classroom observations conducted by project researchers suggested that most QUASAR teachers were successful in identifying and setting up challenging instructional tasks. Nearly three-fourths of the observed and coded instructional tasks placed high-level cognitive demands on students (Stein et al., 1996). These same observations, however, showed that simply selecting and beginning a lesson with a high-level task did not guarantee that students would actually think and reason in cognitively complex ways. In fact, only about 40% of the tasks that started out at a high-level remained that way as the students actually engaged with them (Stein et al., 1996). A variety of factors were found to conspire to reduce the level of cognitive demand of a task once it was unleashed into the classroom environment. The difficulty of maintaining the cognitive demands of a task during implementation was also a key finding of the TIMSS video analysis (Stigler & Hiebert, 2004). In that analysis, none of the tasks that were set up at a high level was implemented as intended.

THE EVOLUTION OF TASKS DURING A LESSON

The fact that tasks take on lives of their own after being introduced into classroom settings has been noted by a variety of classroom researchers (Doyle, 1988; Doyle & Carter, 1984; Stein et al., 1996). In fact, if one wishes to examine task use in the classroom, a reconceptualization of the term task is in order. As mathematical tasks are enacted in classroom settings, they become intertwined with the goals, intentions, actions, and interactions of teachers and students. Therefore, we have found the need to conceptualize mathematical instructional tasks as not only the problems written in a textbook or a teacher's lesson plan (the focus of Chapter 1), but also the classroom activity that surrounds the way in which those problems are set up and actually carried out by teachers and students. Defined in this way, mathematical instructional tasks become situated squarely in the interactions of teaching and learning. When tasks are conceptualized as classroom-based activity, it is not unusual for their cognitive demands to change as they unfold during a lesson. The Mathematical Tasks Framework that was introduced earlier is a visual representation that summarizes the unfolding of tasks in response to the dynamics of teaching and learning in the classroom (see Figure I.3).

The first phase—tasks as they appear in curricular or instructional materials—was discussed in detail in Chapter 1. In this chapter, our focus is on the setup and implementation phases. The setup phase includes the teacher's communication to students regarding what they are expected to do, how they are expected to do it, and with what resources. The teacher's setup of a task can be as brief as asking students' attention to a task that appears on the blackboard and telling them to start working on it. Or it can be as long and involved as discussing how students should work on the problem in small groups, working through a sample problem, and discussing the forms of solutions that will be acceptable.

It is not unusual for a teacher to alter the cognitive demands of the task as she is setting it up for her class. In other words, she may, either purposefully or unwittingly, change the task from how it appeared in the curricular or instructional print materials from which she originally took her idea. For example, consider the Fencing Task, which appeared in the Introduction (Figure I.2). A teacher who thinks that her students are not ready for such an open-ended problem might prepare a worksheet to guide them systematically through a set of solution steps. This worksheet might include the formulas for area and perimeter and a partially completed table that would "lead" students to the discovery that as the pen dimensions approached a square, the area approached its maximum value. The use of this worksheet would take away the challenge introduced by the unstructured nature of the task and hence change its cognitive demands.

The implementation phase (also referred to as the enactment phase) starts as soon as the students begin to work on the task and continues until the teacher and students turn their attention to a new mathematical task. During the implementation phase, both students and the teacher are viewed as important contributors to how the task is carried out. Although the students' levels of cognitive engagement ultimately determine what is learned, the ways and extent to which the teacher supports students' thinking and reasoning is a crucial ingredient in the ultimate fate of high-level tasks (Fenningsen & Stein, 1997; Stein et al., 1996). For example, teachers can promote sense-making and deeper levels of understanding by consistently asking students to explain how
they are thinking about the task. Or, conversely, they can cut off opportunities for sense-making by hurrying students through the tasks, thereby not allowing the time to grapple with perplexing ideas. (See Figure 2.1 for the variety of ways in which students' thinking can be supported or hindered.)

FIGURE 2.1. Factors associated with maintenance and decline of high-level demands (Stein & Smith, 1998).

<table>
<thead>
<tr>
<th>Factors Associated with the Decline of High-Level Cognitive Demands</th>
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<tbody>
<tr>
<td>1. Problematic aspects of the task become routinized (e.g., students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps or perform; the teacher “takes over” the thinking and reasoning and tells students how to do the problem).</td>
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<td>2. The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer.</td>
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<td>3. Not enough time is provided to wrestle with the demanding aspects of the task or too much time is allowed and students drift into off-task behavior.</td>
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<td>4. Classroom management problems prevent sustained engagement in high-level cognitive activities.</td>
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<td>5. Inappropriateness of task for a given group of students (e.g., students do not engage in high-level cognitive activities due to lack of interest, motivation, or prior knowledge needed to perform; task expectations not clear enough to put students in the right cognitive space).</td>
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<td>6. Students are not held accountable for high-level products or processes (e.g., although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are giving the impression that their work will not “count” toward a grade).</td>
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<th>Factors Associated with the Maintenance of High-Level Cognitive Demands</th>
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<tbody>
<tr>
<td>1. Scaffolding of student thinking and reasoning.</td>
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<td>2. Students are provided with means of monitoring their own progress.</td>
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<td>3. Teacher or capable students model high-level performance.</td>
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<td>4. Sustained press for justifications, explanations, and/or meanings through teacher questioning, comments, and/or feedback.</td>
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<td>5. Tasks build on students’ prior knowledge.</td>
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<td>6. Teacher draws frequent conceptual connections.</td>
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<td>7. Sufficient time to explore (not too little, not too much).</td>
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During the implementation phase, the cognitive demands of high-level tasks can easily transform, usually into less-demanding forms of student thinking. The ways in which cognitively challenging tasks typically transform during the implementation phase is discussed in depth in the next section of this chapter.

The ultimate reason for focusing on instructional tasks is to influence student learning (see final triangle in Figure 1.3). Research has demonstrated that the cognitive demands of mathematical instructional tasks are related to the level and kind of student learning. Within the QUASAR Project, students who performed best on the QUASAR Cognitive Assessment Instrument were in classrooms in which tasks were more likely to be set up and implemented at high levels of cognitive demand (Stein & Lane, 1996; Stein et al., 1997). For these students, having the opportunity to work on challenging tasks in a supportive classroom environment translated into substantial learning gains on an instrument specifically designed to measure student thinking, reasoning, problem solving, and communication. Similarly, recent research conducted by Boaler and Staples (2008) suggests that the success of students at Railside (one of the high schools participating in their study) was due in large part to the high cognitive demand of the curriculum and the teachers’ ability to maintain the level of demand during enactment through questioning. Together, these findings suggest the importance of being mindful, both at the outset and during the various task phases, of the kinds of cognitive activity with which students should be and actually are engaged in the classroom.

**PATTERNS OF TASK SETUP AND IMPLEMENTATION**

In a 3-year study of classroom instruction at four QUASAR middle schools (Henningsen & Stein, 1997; Stein et al., 1996), a handful of patterns emerged that captured characteristic ways in which high-level tasks unfolded during instruction. These patterns and the classroom-based factors associated with them are described below.

**Maintenance of High-Level Cognitive Demands**

Some tasks that were set up to place high levels of cognitive demand on student thinking were indeed implemented in such a way that students thought and reasoned in complex and meaningful ways. Take, for example, what happened in Ms. Fox’s class when students were introduced to the Fencing Task (Figure 1.2). Students started out by describing an assortment of pen configurations that could be built with 24 feet of fencing. As they kept coming up with new configurations, they realized they needed to keep track of the shapes they had already tried. This led them to construct a table that
identified the dimensions of each configuration along with its area. Eventually, by looking for patterns across many configurations, students arrived at a conjecture regarding the shape that produced the largest area, and then tested that conjecture with a different amount of fencing (i.e., a different perimeter). During this time, Ms. Fox circulated among the groups, asking questions such as “How do you know you have all of the possible pen configurations?” “Which has the most room?”, and “Do you see a pattern?” These questions led students to see the need to organize their data, make conjectures, and test them out.

Indeed, throughout our data, when tasks were enacted in this way there were usually a large number of support factors present in the classroom environment. As shown in Figure 2.1, these included the selection of tasks that built on students’ prior knowledge, appropriate teacher scaffolding of student thinking (i.e., assisting student thinking by asking thought-provoking questions that preserve task complexity), sustained pressure for explanation and meaning, and the modeling of high-level thinking and reasoning by the teacher or more capable peers.

Other tasks that were set up to place high levels of cognitive demand on students’ thinking, however, exhibited declines in terms of how students actually went about working on them. When the cognitive demands of tasks declined during the implementation phase, a different set of factors tended to be operating in the classroom environment (see Figure 2.1). These factors involved a variety of teacher-, student-, and task-related conditions, actions, and norms. Tasks that declined during the implementation phase generally transformed into one of the forms of student cognitive activity described below.

Decline into Procedures Without Connection to Meaning

Instead of engaging deeply and meaningfully with the mathematics, students ended up utilizing a more procedural, often mechanical and shallow, approach to the task. In this type of decline, one of the most prevalent factors operating in the environment was teachers’ “taking over” and doing the challenging aspects of the tasks for the students.

For example, shortly after Ms. Jones gave her seventh-grade students the Fencing Task (Figure I.2), she was dismayed to see that they were not making much progress—some students were already off-task and many others were complaining that the task was too difficult. Not knowing where to begin, the students began to urge her to give them some help. Wanting them to feel successful and stay engaged, Ms. Jones pointed out to the students that the problem involved finding the area of all the rectangles that had a perimeter of 24. She told her students that they needed to make a chart of all possibilities, starting with a $1 \times 11$, and then find the area for each using the formula $area = length \times width$. Although Ms. Jones’s actions were well intended (and understandable), when she provided students with a procedure for solving the problem, students’ opportunities for mathematical thinking were diminished.

High-level tasks (such as the Fencing Task) tend to be less structured, more difficult, and longer than the kinds of tasks to which students are typically exposed. Students often perceive these types of tasks as ambiguous and/or risky because it is not apparent what they should do, how they should do it, and how their work will be evaluated (Doyle, 1988; Romagnano, 1994). In order to deal with the discomfort that surrounds this uncertainty, students often urge teachers to make these types of tasks more explicit by breaking them down into smaller steps, specifying exact procedures to be followed, or actually doing parts of the task for them. When the teacher gives in to such requests, the challenging, sense-making aspects of the task are reduced or eliminated, and the opportunity to develop thinking and reasoning skills and meaningful mathematical understandings is lost.

Decline into Unsystematic Exploration

Unsystematic exploration differs from the other categories previously discussed since it is not used to describe tasks as they appear in curricular materials or as they are set up by the teacher and it was not represented in the QUASAR researchers’ original coding scheme. Rather, this category emerged from the analysis as a way to describe some doing-mathematics tasks that were not proceduralized but still not adequately implemented. In this type of decline, students approached the task seriously and attempted to perform mathematical processes such as conjecturing, looking for patterns, discussing and justifying, and so forth. However, they failed to progress toward understanding the important mathematical ideas embodied in the tasks. Take, for example, Mr. Chambers’s experience with the Fencing Task. Although his seventh-grade students worked conscientiously during the entire period, they focused on aspects of the problem (e.g., How big were the rabbits? How much space did the rabbits need? How much would the fencing cost?) that were not central to answering the questions posed. Although students’ thinking required decision making and involved some mathematics, it did not move the students toward the generalization that the largest area for a fixed perimeter would be a square—the point of the task.

In cases such as that of Mr. Chambers, teachers appeared to desire to maintain the complexity of the task; they usually didn’t take over and/or
oversimplify tasks. But they also did not provide the kind and the extent of support that teachers provided when high levels of cognitive activity were maintained. For example, the sensitive, thought-provoking questions that Ms. Fox interjected at crucial points of students’ explorations were absent. Another factor that seemed to be associated with this pattern was too much time for students to work on the task; without needed supports they floundered, failing to make progress toward mathematical understanding.

**Decline into Nonmathematical Activity**

In these cases, students often displayed a variety of off-task behaviors such as playing absentmindedly with their manipulatives or talking with their partners about subjects far afield from mathematics. This often happened when the task was not matched appropriately to students’ prior learning experiences and/or expectations were not specific enough to guide students into an appropriate mathematical space. Another factor that played a significant role in this type of decline (to a much greater extent than in other kinds of decline) was classroom-management problems. When students were free to roam the room and talk with friends during group-work time, or to disrupt the class with requests for materials, then all students’ abilities to engage with complex tasks were sacrificed.

Tasks can also decline into no mathematical activity when the teacher does not keep the focus on mathematics. In these situations, students are engaged in activity, but the activity tends to be nonmathematical in nature. For example, when Ms. Jackson used the Fencing Task in her class, she asked each student group to produce a poster on a large sheet of newsprint, showing their work in an organized way. The students’ attention turned immediately to the creation of posters as works of art rather than as the result of mathematical activity, producing elaborate drawings of rabbits and pens and titling their work in calligraphy. In this situation, the teacher failed to keep an eye on the mathematics, settling instead for more affective outcomes such as students’ working well together.

These four patterns—all of which began with a task classified as *doing mathematics*—represent a subset of the most prevalent patterns of task setup and implementation identified in our research. All of the prevalent patterns are identified in Figure 2.2. As shown in the figure, each pattern has been found to be associated with a set of classroom-based factors that appear to influence the path of task evolution. It is interesting to note that when the level of cognitive demand is maintained, the same five factors are generally present. When tasks decline, however, the set of factors varies depending on the nature of the decline.

![Using Cognitively Complex Tasks in the Classroom](Figure 2.2. Common patterns of task setup and implementation and most-frequently associated factors. For each pattern, the factors are ordered from most- to least-frequently observed.)

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**Factors Most Often Associated with Specific Patterns of Maintenance and Decline**

<table>
<thead>
<tr>
<th>Patterns</th>
<th>Task Setup</th>
<th>Task Implementation</th>
<th>High-Level Demands</th>
<th>Factors Most Often Associated with Specific Patterns of Maintenance and Decline</th>
</tr>
</thead>
</table>
| Doing mathematics → Doing mathematics | Maintained | - Task builds on students’ prior knowledge  
- Scaffolding  
- Appropriate amount of time  
- High-level performance modeled  
- Sustained pressure for explanation and meaning |
| Doing mathematics → Procedures without meaningful connections | Declined | - Challenges become nonproblems  
- Focus shifts to correct answer  
- Too much or too little time |
| Doing mathematics → Unsystematic exploration | Declined | - Inappropriateness of task for students  
- Too much or too little time  
- Challenges become nonproblems |
| Doing mathematics → No mathematical activity | Declined | - Inappropriateness of task for students  
- Classroom management problems  
- Too much or too little time |
| Procedures with connections → Procedures with connections | Maintained | - Task builds on students’ prior knowledge  
- High-level performance modeled  
- Appropriate amount of time  
- Sustained press for explanation and meaning  
- Scaffolding |
| Procedures with connections → Procedures without connections | Declined | - Challenges become nonproblems  
- Focus shifts to correct answer  
- Inappropriateness of the task for students |
In Part II of this book, we present cases of classroom instruction that exemplify each of these six patterns (shown in Figure 2.2) and the factors associated with them. It is our hope that these cases will help teachers identify when and why various patterns occur in a more self-conscious and consistent manner than they might normally do on their own. This will, in turn, help teachers become more alert to the potential for slippage between intention and action in their teaching. The self-monitoring and reflection required to analyze instruction in this way represents an important first step toward providing students with the enhanced learning opportunities they need in order to develop into powerful mathematical thinkers.

Before introducing the cases, we conclude Part I of the book with a chapter that provides an overview of our theory of how teachers learn from cases.

NOTES

1. In our framework, task implementation refers to the enactment of a task in the classroom by teachers and students. In our more current writings, including this book, we frequently use the term enactment as a synonym for the term implementation. Enactment appears to avoid the sometimes negative connotations associated with implementation (i.e., implementation as mindless performance of mandated practices), and the misinterpretation that teachers can be thought about only as implementors rather than as constructors. By contrast, enactment connotes the view that instructional tasks are co-constructed through the thoughts and actions of the teacher and her students during the course of instruction. We have continued to label this phase “tasks as implemented by students” in the framework, however, in order to preserve continuity with past publications.

2. For additional information on the QUASAR Cognitive Assessment Instrument, see Lane (1993), Lane and Silver (1995), and Silver and Lane (1993).

3. The two patterns not discussed in the preceding paragraphs involve tasks that were set up as procedures with connections (see the final two patterns in the figure).

CHAPTER 3

Learning from Cases

Although the professions of business, law, and medicine have used cases for decades to teach the main ideas, skills, and underlying principles of their practice, the use of cases in teacher education is relatively recent. After Lee Shulman’s 1985 American Educational Research Association (AERA) presidential address in which he called for the development of a case knowledge of teaching, a variety of practitioners and researchers began writing, using, and studying cases as tools for educating teachers. The use of the case method can now be observed in a variety of teacher-education and staff-development programs across the country.

THEORETICAL CONSIDERATIONS

Professional users of the case method are often asked what teachers learn from cases and how they learn it. Their answers vary, often in relationship to the kind of case being used. For example, dilemma-driven cases close with a pedagogical problem to be solved (see, for example, cases edited by Barnett, Goldenstein, & Jackson, 1994). Exposure to these kinds of cases aims to help teachers (1) realize that teaching is an inherently dilemma-ridden enterprise and (2) learn how to think about the trade-offs involved in selecting one course of action over another.

The cases in this book are of a different kind. We think of them as paradigm cases (Shulman, 1992), that is, cases that embody certain principles or ideas related to the teaching and learning of mathematics. Exposure to our cases aims to assist teachers to develop (1) an understanding of mathematical tasks and how their cognitive demands evolve during a lesson and (2) the skill of critical reflection on their own practice guided by reference to a framework based on these ideas.

Our ultimate goal is to influence instructional practice through teacher reflection. Teachers often report that, without guidance, it is difficult to get a handle on how to think about their own instruction. For example, when teachers first view a video of their own practice, they often lack a coherent focus and thus experience reflection as a frustrating attempt to decipher and bring