Chapter 2

Classroom Practices

To be formative, assessment must include a recipe for future action.
—Dylan Wiliam

PRINCIPLES and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM] 2000) identifies both Content Standards and Process Standards. The Process Standards require students to “discuss and validate their mathematical thinking, create and analyze a variety of representations that illuminate the connections within the mathematics, and apply the mathematics that they are learning in solving problems, judging claims, and making decisions” (NCTM 2006, p. 8).

NCTM also developed Curriculum Focal Points to focus instruction in each grade level, K–8, on three key mathematical ideas. Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (NCTM 2006) further develops the three key ideas for each grade level through connections to other identified mathematical strands. Curriculum Focal Points locates the mathematics for each grade firmly in contexts that promote the Process Standards: problem solving, reasoning, communication, making connections, and designing and analyzing representations.

The Common Core State Standards (Common Core State Standards Initiative [CCSSI] 2008) were developed to standardize mathematical practices and learning standards across the country. The National Governors Association in conjunction with the Council of Chief State School Officers oversaw this national initiative. The standards articulate grade-level learning goals, as well as different pathways of focus for high school students.

Implicit in both Principles and Standards and Curriculum Focal Points is that important concepts should be taught deeply and taught well. Students are expected to think—to be engaged in solving problems that address grade-level topics at high levels of cognitive demand.

Crucial to such instruction is teachers’ knowledge of what students know about the concept to be taught. Using formative assessment in the classroom is a natural extension for understanding what students know and can do, as well as how well they can communicate mathematics verbally and symbolically. The nine formative assessment strategies and tools in this chapter allow you to gather quick but accurate information about your students’ understandings, misconceptions, and challenges linked to particular mathematical goals. Figure 2.1 aligns the strategies with their assessment goals.
Fig. 2.1. Aligning strategies with assessment goals

**Toolkit for Formative Assessment**

Identifying tools that make assessment for learning practical is a vital step in implementing daily formative assessments. Formative assessment, when well designed and implemented, enables you to develop instructional activities for the specific needs of your students, to make adjustments as students progress through a unit of study, and to give all students appropriately challenging mathematical tasks. Unlike summative assessment, formative assessment is often “invisible” to the students because it is part of daily classroom interactions, is frequent and quick, takes place while students are working, and does not result in a grade.

The nine classroom strategies and tools we describe here allow you to gather evidence about
your students’ understandings, misconceptions, and challenges in an interactive, low-stress setting. Many of the practices require you to think differently about how you engage your students in doing mathematics and how you determine daily instructional goals and activities. Teachers have implemented each identified practice in classrooms, and both teachers and students found the practices effective. We highlight nine formative assessment tools:

1. Range questions
2. Observation protocols
3. Gallery walks
4. Round-robin activities
5. Focused questions and hinge questions
6. Interviews
7. Mathematical discourse—accountable talk and math congress
8. Neutral feedback
9. Exit cards (exit slips)

**Gather Evidence about Students’ Understanding**

Our overarching goal is to engage students in work that will help them develop proficiency in mathematics. Students who struggle to understand the relevance of mathematics in the world and in their own lives—constantly asking “why are we doing this?”—tend to be doing worksheet- and textbook-driven mathematics focused on procedural skills with low levels of cognitive demand. Assessments based on the worksheet model produce evidence of students’ ability to complete worksheets and problems from textbooks. Students who have adapted to a worksheet-driven instructional model can test well on procedural skills yet still not have a deep understanding of concepts.

Conversely, in classrooms where teachers assign mathematically rich, interesting problems and encourage student interactions around mathematics, students challenge each other while modeling mathematics, making predictions, formulating conjectures, and experimenting with physical data. They represent those data in tables, charts, or graphs, complete with appropriate equations or justifications. These students are doing math: using procedural skills in the service of mathematics with a high level of cognitive demand. They work in the realm of process skills, using higher-order thinking skills as they work on complex mathematical tasks at the appropriate level of challenge. In this model, you must continually gather evidence of students’ understandings to ensure that each student meets the curricular standards and understands the essential concepts.

The interactions among students as they do math may result in a classroom that does not conform to the traditional image of a mathematics classroom, but without interaction, the level of cognitive demand will remain low. The formative assessment strategies and tools we discuss are
well suited to the latter instructional model and will facilitate an “orderly chaos” in which you will be able to gather evidence about students’ performance.

**Assessing prior knowledge: Range questions**

Awareness of what a child knows and understands is the first step in planning instruction designed to deepen that knowledge and understanding. Before assigning a problem, you must identify the standard or focal point with which the item is aligned. Thinking reflectively about the item’s pivotal concept is also appropriate. You also need to know about the key misconceptions and challenges that many students share and to consider whether the question presents a low, medium, or high level of cognitive demand.

After reflecting on those components, you are ready to identify questions to ask before the lesson begins to assess students’ prior knowledge. Range questions (Heritage 2008) are posed at the beginning of a lesson to determine students’ scope of understanding. These questions are designed to be quick—no more than five minutes—and you can post them in the classroom as students enter or as they begin their mathematics lesson. Range questions should be relevant to that day’s lesson concept. Most important, range questions are starting points for solving problems and often lead to pivotal understandings. Sample range questions include the following:

- Name two fractions that have a value greater than $\frac{2}{7}$.
- Name two fractions that have a value less than $\frac{2}{7}$.
- Tell me what you see: $\frac{3}{4}$
- Turn to your partner and tell him or her everything you can about $y = -3x + 4$.
- In what order would you place the following fractions on a number line? $\frac{3}{4}, \frac{5}{8}, \frac{2}{5}, \frac{1}{2}$
- Show me about how long 5 centimeters is.
- What do you know about the multiples of 6?
- If you collect a penny a day, how long do you think it will take to collect a million pennies? A billion pennies?

These questions are broad, allow for more than one answer, and require students to share their thinking.

Part of the planning for any lesson involves identifying a problem that you can adjust to meet the needs of all students. Students who answer the range question with strong mathematical reasoning need a problem that enriches their knowledge, deepens their understanding, and moves them forward in their learning. Students who do not answer correctly or who exhibit uncertainty about their response may require a version of the problem designed to support and solidify their knowledge. Still others may need more scaffolding to begin to think about the same problem. This scaffolding could be as simple as breaking down components of the problem by using bullets to help the students focus on one part of the problem at a time.

Range questions with specific tasks for each grade are in chapters 3–5. You can find more generic range questions in Marian Small and Amy Lin’s 2010 book *More Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction*. 
Assessing students at work: Observation protocols

As anyone who has taught middle school knows, students at this age want to be physically active and talk with their classmates. In fact, research suggests that they need to move around and converse with one another. Classrooms with successful students routinely have students who are out of their seats modeling mathematics, making predictions, formulating conjectures, and experimenting with physical data—which they then organize in tables, charts, or graphs, complete with appropriate equations or justifications.

As long as you establish norms and expectations for behavior, you can harness and incorporate into daily classroom routines this need for movement and interaction, which makes a traditional teaching style difficult. Teachers who involve their students in developing classroom protocols complete with posted rubrics detailing classroom expectations typically have fewer problems with classroom management and behavior.

You can use one such protocol, observation, to gather evidence about students’ proficiency in problem solving, computation, and communication, as well as about their disposition toward mathematics. This protocol’s focus depends on the criteria you set for your students. If, while your students are working collaboratively, they must share problem-solving strategies, communicate orally and symbolically, compute with or without a calculator, justify their answers, or report on their findings, you can design the observation protocol to record each student’s level of competence in those areas. If you are concerned about knowing when a particular student gets frustrated and gives up, you can include that category in the protocol and keep a record of whether the student’s tolerance improves as he or she becomes more confident with the mathematics.

Observational protocols, when well designed and carefully recorded, serve as records of students’ progress and give you evidence on which to base instructional decisions.

Practicing teachers wrote the protocols in figures 2.2 and 2.3 to document their students’ mathematics proficiency, habits of mind, and mathematical confidence. Each reflects the teacher’s understanding of the many elements of mathematical proficiency. The teachers used the completed forms as evidence when interviewing their students. Another teacher found the observational protocol in figure 2.3 better suited to her teaching style.

Assessing students at work: Gallery walks

Sharing students’ work is an important component of a community of learners. Gallery walks offer students a forum in which they can rotate among various stations where they can post a comment, strategy, question, or solution. While students are working their way through the stations, you listen for appropriate mathematical terminology, listen for mathematical arguments, and watch for appropriate symbolic notation. If, for example, students are working on a geometry unit that requires understanding geometric vocabulary, you might design a gallery walk with eight or ten stations. Each station could have a geometric term written on large poster paper. Students, in small groups, rotate among the stations, posting their definition of each term. A class discussion may follow that compares the definitions and lets students agree on an accurate mathematical definition.
<table>
<thead>
<tr>
<th>Concept</th>
<th>Behaviors and effort</th>
<th>Student habits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student name</td>
<td>Group work</td>
<td>Learning and understanding</td>
</tr>
<tr>
<td>Takes notes</td>
<td>Active participant</td>
<td>On task</td>
</tr>
<tr>
<td>Answers questions</td>
<td>Helps others</td>
<td>Completes work</td>
</tr>
<tr>
<td>Asks topical questions</td>
<td>Copies work from others</td>
<td>Accountable talk</td>
</tr>
<tr>
<td>On task</td>
<td>Helps</td>
<td>Help</td>
</tr>
</tbody>
</table>

Fig. 2.2. Sample observational protocol

<table>
<thead>
<tr>
<th>Skill</th>
<th>Opening activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achieve learner</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2.3. Another sample observational protocol
A well-organized gallery walk promotes critical thinking; written expression; oral communication; and an interactive, student-centered environment. Assign students problems that require them to analyze, predict, compare, construct, or justify so that, after they complete the mathematics, a rich discussion can follow. After posting their work, students should expect to justify their thinking, field questions from their classmates, and make connections between their work and previous mathematical experiences—all features of mathematical tasks at a high level of cognitive demand.

Gallery walks can serve as formative assessments, with teachers using an observation protocol to record students’ involvement. You can also use gallery walks more formally as summative assessments by posting rubrics before students begin the process. You must be clear on the gallery walk’s purpose before beginning one. If the gallery walk is designed as formative assessment, you should listen to students as they discuss their work. If you have designed the gallery walk as summative assessment, in which students are graded, students must know in advance about how you will grade the work. Many teachers find that involving the students in developing the rubrics that identify exemplary work is most effective.

Gallery walks have the flexibility of working effectively in as little as fifteen minutes or to allow a forum in which students may present a unit project over days or weeks. The gallery walk at the end of a unit might showcase students’ culminating activity. It may include a collage to exhibit students’ favorite numbers and all the mathematical information they learned about them, or it may include illustrations of exponential growth or decay, or the design for a community playground (Lappan 2006). Such a gallery walk may be timed to coincide with parent–teacher conferences, family math nights, or open houses to allow students to display their work to an audience outside their classroom.

Students can also assess the success of the gallery walk. You can use the students’ reflection evaluation in figure 2.4 to determine how well you structured the gallery walk.

<table>
<thead>
<tr>
<th>General Observation</th>
<th>Disagree</th>
<th>Neutral</th>
<th>Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 The directions for the gallery walk were clear. I knew what to successfully complete a gallery walk.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 The topics in the gallery walk were interesting to me.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 We worked more collaboratively in the gallery walk than we do with usual class discussion techniques.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 During the gallery walk, all group members participated and listened respectfully to one another.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2.4. A gallery walk evaluation form for students—Continues
One Monday, a teacher concerned about the potential chaos of a gallery walk decided to be adventurous. She assigned a gallery walk to gather evidence about her students’ understanding of mathematical vocabulary terms. She posted the mathematical terms profit, income, and expense. After reading the students’ definitions, she realized that most of her students truly had not mastered these terms’ meanings. She spent the week working on those concepts, through problem solving, and on Friday revisited the terms in a gallery run. A gallery run operates the same way as the gallery walk, except that it gives students only half the time to complete the activity. Figures 2.5–7 show this gallery run’s results. The teacher was pleasantly surprised with the progress her students displayed, but she also documented that some misconceptions remained.

Fig. 2.4. A gallery walk evaluation form for students—Continued

<table>
<thead>
<tr>
<th>General Observation</th>
<th>Disagree</th>
<th>Neutral</th>
<th>Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 I felt I gained a better understanding of the topic if I learned the topic through lecture. (Explain your answer.)</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 The wording of gallery walk questions was clear. If not, which questions needed improvement? Question 1 2 3 4 5</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 I felt we had enough time to discuss each topic at learning stations.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 The gallery walk was easy to use.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 The evaluation criteria (how I will be graded) for the gallery walk were clear.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 My overall experience with the gallery walk was satisfactory.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 I would like to participate in another gallery walk.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2.5. Students’ definitions of income from the gallery run
Most groups appear to understand the meaning of income; however, two posts appear to offer evidence of confusion between profit and income. Some groups described income as money received rather than earned, or money you get but no indication of how you get it. These statements warrant further discussion.

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**Fig. 2.6. Students’ definitions of expense from the gallery run**

Most students defined expense as an amount of money spent on certain goods. Yet one group of students referred to an amount of money “lost”—this comment needs further discussion. Also, notice the last comment: “The amount of money you earn.” This group of students misunderstands expense, and further discussion is appropriate.

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**Fig. 2.7. Students’ definitions of profit from the gallery run**

Students also varied in their definitions of profit. Some evident conceptual misunderstandings include defining profit as “expenses” and “the money you are making.”

Going by the results posted in the gallery run, the teacher decided to continue including problems involving profit, expense, and income in her range questions, exit cards, and homework assignments. She also decided that the gallery walk was so successful that she would use it to share students’ problem solving.
Assessing students at work: Round-robin activities

Middle school students are expected to master the order of operations, simplifying expressions, and solving multistep equations, among other procedural skills that both Curriculum Focal Points and Principles and Standards identify. Students need time to develop proficiency with these mathematical skills and procedures as well as with conceptual understanding. One lesson with homework will not be enough to develop deep understanding for any student. Engaging students in interactive activities allows them to explore ideas from varied entry points and mathematical preparation, holds their interest longer, and is more effective than assigning worksheets.

The round-robin activity is designed to allow you to assess students while they work. It also lets you intervene as needed before any misconception or procedural misstep can germinate.

Here’s how you run a round robin. Group students into threes, with each group member assigned the number 1, 2, or 3. Instruct each student who is a number 1 to go to the board. Dictate an expression or equation for that student to record. On completion of the dictation, student number 1 does one step and then sits down. Student 2 goes up to the board, does step 2, and then sits down. Student 3 follows student 2 to the board and does step 3. This process continues until the computation is complete (fig. 2.8).

<table>
<thead>
<tr>
<th>All students assigned the number 1 go to the board and write:</th>
<th>( 16 - 3(17 - 4 \times 3) + 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1 does one step:</td>
<td>( 16 - 3(17 - 12) + 5 )</td>
</tr>
<tr>
<td>Student 2 takes over and does one step:</td>
<td>( 16 - 3(5) + 5 )</td>
</tr>
<tr>
<td>Student 3 takes over and does one step:</td>
<td>( 16 - 15 + 5 )</td>
</tr>
<tr>
<td>Student 1 returns and does one step:</td>
<td>( 1 + 5 )</td>
</tr>
<tr>
<td>Student 2 completes the computation:</td>
<td>( 6 )</td>
</tr>
</tbody>
</table>

Fig. 2.8. The round-robin approach

The formative assessment component is a seamless part of the activity. Observe each group at the board. If a student erred in the first step and wrote \( 13(17 - 4 \times 3) + 5 \), you would sidle up to the student and quietly ask where the 13 came from. You might ask the student to list the order in which the operations are done or to name the terms in the problem. You may highlight that the 3 is a factor and ask what the second factor is. The important part is ensuring that the student does not make a conceptual error without immediate intervention. Often during the round robin, students help their teammates if they see an error, so the formative assessment is about peers helping peers.

After one computation is complete, instruct the students with the number 2 to go to the board. Dictate another problem, and have student 2 do the first step, followed by student 3, and then student 1.

For solving multistep equations, ask the students not only to complete one step of the procedure but also to identify the mathematical property the student used (fig. 2.9)
You dictate $3a + 7 - 5a = 2(4a - 3) + 2a$.
The first student may record $7 - 2a = 2(4a - 3) + 2a$ (combine like terms).
The next student may record $7 - 2a = 8a - 6 + 2a$ (distributive property).
The next student may record $7 - 2a = 10a - 6$ (combine like terms).
The next student may record $7 - 2a + 2a = 10a - 6 + 2a$ (additive inverse).
The next student may record $7 + 6 = 12a - 6 + 6$ (additive inverse).
The next student may record $13 ÷ 12 = 12a ÷ 12$ (multiplicative inverse).
The next student may record $\frac{13}{12} = a$.

**Fig. 2.9. The round robin for a multistep problem**

The round-robin technique offers several advantages:

- It shows how well students can enter a problem-solving process at various decision points.
- It actively engages all students in problem solving.
- It fosters collaborative work among students’ peers.
- It gives the class the opportunity to work through many different problems within a short period.
- It reinforces students’ skills in a nonthreatening environment.

**Probing students’ thinking: Focused questions**

Think for a moment about how you ask your students a question during a whole-class discussion. Do you pose a question, call on one student to answer the question, and then move on after the student answers or you get a correct answer? This typical method of questioning is actually a conversation between two people—you and the student asked to respond. Students not involved in the conversation don’t have to think.

Or do you pose a question, directing your students to think about the question individually for a minute or two before instructing them to turn to their partner and discuss the question? After a specified wait time, do you ask a representative from each pair or small group to report their responses? This method of questioning is often referred to as *active listening* or *think-pair share*. Such an approach engages all students. All students think about a question, discuss the question with their peers, and finally report their responses.

A conjecture is similar to a hypothesis in science: an idea that is tested, revised, and retested. The teacher often records on a conjecture board the various responses students offer from range questions, problems, or suggested algorithms. Think of the conjecture board as a parking lot—a location designated for recording student ideas. As students work through their mathematics lesson, teachers encourage them to think about whether the conjecture is always mathematically true, test the conjecture, revise it when and if the original conjecture is inaccurate, remove it if
someone gives a counterexample or a negation, and then test the revised conjecture. One teacher reports that her students worked harder to find a negation or counterexample to the posted conjectures than they did on their assigned work. She includes the conjecture board in her daily lessons.

**Probing students’ thinking: Hinge questions**

Bright and Joyner (2005) discuss three different types of questions to ask students when they are working individually or in pairs or small groups: engaging questions, clarifying questions, and refocusing questions.

Heritage (2008) refers to hinge questions in addition to the earlier range questions. Hinge questions depend on what students are saying or doing. Similar to Bright and Joyner’s conceptual model, hinge questions might involve engaging, refocusing, or clarifying questions. The type of question teachers ask depends on what is happening with the students at various times throughout the lesson.

Even so, you must anticipate the questions you think might arise and have a small battery of relevant engaging, refocusing, and clarifying questions ready before you ask students to work the problem. The best questions take thought. You might come up with one spontaneously, but a little planning will give you a better chance to get the most out of the teachable moment when it arises.

**Engaging questions**

Engaging questions are fairly easy to ask if students are working on an interesting problem. These questions are designed to engage students with the problem. You can ask engaging questions during the problem-solving process when a student stops participating and needs a nudge to return to work on the assigned problem. The problem or task should be interesting and appropriate for the particular grade level. A well-thought-out engaging question should spark students’ interest and make them want to solve the problem.

The following problem often intrigues middle school students and lends itself to asking engaging, refocusing, and clarifying questions: *If you begin counting your heartbeats at exactly twelve o’clock New Year’s Day, when will your heart have beaten 1 million times? Support your answer.* Think about this problem. It’s a straightforward problem that interests most students. Why is it interesting? What are the mathematical concepts behind it? What information do your students need to consider? How might your students begin? Would you ask your students to make a prediction before you instructed them to work on finding a solution? Would you invite students to work in pairs or small groups to solve the problem?

Some engaging questions appropriate for this problem include the following: *Do you think each member of your group has the same heart rate? How can you determine each person’s heart rate? How can you incorporate each person’s heart rate in your solution?* These questions are designed to guide students toward thinking about taking one another’s pulse and using the data to find an average. If students did determine the mean heart rate, two follow-up engaging questions might be *What would happen if you used the median class heart rate? and How would your results differ?*

Now, compare the question about heart rates to a more typical question students are asked,
which includes the same basic mathematical processes:  \( \text{Compute } 1,000,000 \div 72. \)

Which question is more likely to appeal to students? More important, which solution process allows you to ask engaging questions and yields evidence about students’ conceptual understanding and procedural skills?

**Refocusing questions**

Refocusing questions prevent students from pursuing a dead-end strategy when solving a particular problem, guide students to answer the question asked, and return them to the task at hand if they drift off. A refocusing question would be appropriate if students are using just one person’s heart rate. For example, *I noticed that you have already started doing calculations that involve multiplication. Can you explain to me what the calculations represent? How will they help you find out how many times a heart beats every hour?*

**Clarifying questions**

Clarifying questions ask students to explain what they are thinking or to clarify the assignment or directions. For this problem, clarifying questions might include the following: *Are you sure everyone in your group has the same heart rate? Do you think your heart rate stays the same all day? How is multiplying your heart rate by 60 seconds in a minute different from dividing your heart rate by 60 seconds in a minute? Which do you think will best help you begin to determine when your heart rate beats the millionth time? What were you thinking when you multiplied 1 million by 60 seconds in a minute? If your heart beats 72 times per minute, how many times will it beat in 2 minutes? 3 minutes? 10 minutes?* Each question focuses on making students’ thinking visible.

**Probing student thinking: Interviews and conferences**

A natural follow-up to the questioning process is the interview. Teachers interview individual students to ensure exact understanding of what students are thinking about their problem solving. Interviews should take place outside the mathematics class time in a quiet area where the student can feel relaxed and comfortable. Setting the environment for the interview is important to reassure students that the interview is not punitive but rather to give the teacher insights about the students’ thinking to better meet their learning needs.

A grade 7 teacher assigned the following problem to her students as a range question: *A photograph that is 6 inches on the base and 8 inches high is to be enlarged so that the new base is to be 15 inches. What will the height of the enlargement be?*

She instructed her students to predict the length of the enlarged photograph without using paper or pencil. She recorded the range of answers on the board. The responses included 2.5, 17, 20, and 25 inches. She decided to interview the students whose answers might have reflected a misunderstanding of proportionality, which she believed the answer of 17 inches indicated. The following is an excerpt from one of those interviews.

*Teacher:* Can you tell me what you were thinking when you predicted 17 inches as the answer?
Student: That was easy. I saw that 15 is 9 more than 6, so I need a base that is 9 more than 8.

By interviewing the student, the teacher confirmed her suspicion that the student was thinking additively rather than multiplicatively, so the teacher planned her next lesson with that knowledge in mind. She decided to challenge her students to build similar figures that had different scale factors by using geometric shapes. She instructed her students to record their findings in a table and to make predictions about the enlargement of the sides in relation to the enlargement of the perimeters versus the areas of the similar figures. This was a lesson she had not originally planned to do, but the interviews showed the necessity of giving her students a more concrete investigation to build conceptual understanding.

**Probing student thinking: Mathematical discourse**

Discourse entails fundamental issues about knowledge: What makes something true or reasonable in mathematics? How can we figure out whether or not something makes sense? That something is true because the teacher or the book says so is the basis for much traditional classroom discourse. Another view, the one put forth here, centers on mathematical reasoning and evidence as the basis for the discourse. In order for students to develop the ability to formulate problems, to explore, conjecture, and reason logically, to evaluate whether something makes sense, classroom discourse must be founded on mathematical evidence. (NCTM 1991, p. 34)

Student-centered mathematics classrooms promote conversation in small groups, between student and teacher, and between student and student. The conversations about mathematical ideas and work are fundamental to learning. Resnick (1999) suggests that classroom talk that promotes learning must have certain characteristics:

- Students seriously respond to and further develop what others in the group have said.
- Students put forth and demand knowledge that is accurate and relevant to the issue under discussion.
- Students use evidence in ways appropriate to mathematics, including the use of proof.

Students at any age can converse about mathematics. The teacher must develop classroom norms to ensure that important mathematical ideas are discussed at a high level of cognitive demand and that all students participate. Facilitating effective classroom discourse is not easy. *Professional Standards for Teaching Mathematics* states the following:

The teacher of mathematics should orchestrate discourse by

- posing questions and tasks that elicit, engage, and challenge each student’s thinking;
- listening carefully to students’ ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students’ ideas;
• deciding when to provide information, when to clarify an issue, when to lead, and when to let a student struggle with a difficulty;
• monitoring students’ participation in discussions and deciding when and how to encourage each student to participate. (NCTM 1991, p. x)

Teachers and educational researchers have developed many models for classroom discourse. We highlight three in this book, but many others exist.

**Accountable talk**

According to Resnick (1999, p. 39), “Accountable talk sharpens students’ thinking by reinforcing their ability to use knowledge appropriately. As such, it helps develop the skills and habits of mind that constitute intelligence-in-practice.”

Accountable talk is conversation in a mathematics classroom that is accountable to community, to accurate knowledge, and to rigorous reasoning. Using accountable talk standards during either whole-class or small-group discussions raises the level of cognitive demand as students’ peers challenge them to explain or prove that their thinking is correct.

In an accountable talk session, students discuss a mathematical idea, make conjectures, and support or negate those conjectures mathematically. The conversations are based on a topic and question that you choose to ensure that students meet their learning goals. Accountable talk in a mathematics classroom is best used when students discuss a mathematics problem with other students. Features of accountable talk include the following:

- Restating what another person said
- Restating in different words what someone said
- Expanding on a previously stated idea
- Asking whether an idea, strategy, or solution makes sense
- Asking for clarification when confused
- Using evidence to support statements by (1) including a mathematically accepted definition, (2) using data gathered from an investigation, or (3) negating a conjecture or observation

Teachers direct the discussion by directing students’ attention to ideas they want the class to consider, but without judging the worth of the ideas. “Talk moves” include the following:

- Revoicing. *So let me see if I’ve got your thinking right. You’re saying ________?*  
  (Follow with time for students to accept or reject the teacher’s formulation.)
- Asking students to restate someone else’s reasoning. *Can you repeat what he just said in your own words?*
- Asking students to apply their own reasoning to someone else’s reasoning. *Do you agree or disagree, and why?*
- Prompting students for further participation. *Would someone like to add on?*
- Asking students to explicate their reasoning. *Why do you think that? or How did you
arrive at that answer? or Say more about that.

- Challenging or offering a counterexample. Is this always true? or Can you think of any examples that would not work?

As you monitor the discussion, you must also listen for appropriate use of mathematical vocabulary. Teachers who model use of appropriate mathematical vocabulary often find that their students begin to use mathematically correct vocabulary in their discussions, when they make and revise conjectures, and when they pose exceptions and negations to earlier conjectures. Use of accountable talk tends to become more precise as students move through middle school into high school and becomes a powerful step in understanding the nuances of mathematical proof.

**Math congress**

A math congress is a forum to discuss one or two big ideas that have developed through student work on a specific problem. After students have spent time investigating a problem, they are asked to work with a partner or two to decide what parts of their work they want to share with classmates. Students prepare for the congress by making a chart that shows ideas and strategies that they want to share or discuss.

“The math congress continues the work of helping children become mathematicians in a mathematics community—it is a forum in which children communicate their ideas, solutions, problems, proofs, and conjectures to each other” (Fosnot 2007, p. 39). Fosnot developed the math congress activity to be used as part of a mathematical investigation in a classroom where instruction takes place in a workshop model. She believes that “we become mathematicians by engaging with mathematical problems, finding ways to mathematize them, and defending our thinking in a mathematical community” (Fosnot 2007, p. 27).

The teacher’s role in a math congress is complex. Keeping in mind the big idea and the learning goals of the class, you must choose only a few pieces of student work to use and guide the discussion so that each student moves forward in mathematical thinking. Your preparation is essential to a powerful learning experience for the class. Fosnot (2007, p. 29) suggests how the conversation might flow:

- What ideas deserve discussion? In what order?
- Can some of the ideas be generalized? How will you promote this?
- Is there a possible sequence in the discussion that might serve as a scaffold to learning?

You may see that several students have represented their work in similar ways and choose to structure the discussion around the similarities and differences of the representations—and whether they would work for other, but similar, types of problems.

In a different situation, you might decide to choose three different computational strategies, directing discussion toward questions of replicability and efficiency. For example, you could present the following problem to students: I have $4\frac{1}{2}$ yards of ribbon. I want to make bows. For each bow, I need 2 feet of ribbon. How many bows can I make? Students will probably use many different strategies to solve this problem. You can use the different strategies to explore various
meanings of division and division of fractions, to describe the meaning of a remainder, and to work on the big idea that division is the inverse of multiplication.

Fosnot continues, “In addition to helping students learn to calculate with greater understanding and capacity, these methods also allow teachers to capitalize on children’s thinking in order to deepen their knowledge of mathematics—a capitalization not available when students are restricted to the traditional method or calculator” (2007, p. 29).

Young Mathematicians at Work: Investigating Decimals, Fractions, and Percents, Grades 4–6, has more suggestions for facilitating a math congress around particular problems.

**Using assessment to plan instruction: Neutral feedback**

“The most powerful single motivator that enhances achievement is feedback” (Hattie 1992).

Most students, when they get work back, immediately search for a grade. If the grade is good, chances are they will save the paper, but if the grade is average or poor, the papers will probably find their way into the trash bin. Students who receive a grade and written feedback usually look at the grade and often disregard the written feedback. But research is showing that if written comments are the only written information on the page, students read the feedback and seriously consider what it says.

Feedback often falls into two categories: judgmental and informative. If we are to have a positive impact on student achievement, we need less judgmental feedback and more informative feedback. Adapted from Black and Wiliam (1998), table 2.1 illustrates the differences between summative and formative feedback and their impacts on students.

Feedback can be categorized as motivational, evaluative, descriptive, or effective.

**Table 2.1—Continues**

**Summative versus formative feedback**

<table>
<thead>
<tr>
<th>Feedback type</th>
<th>Characteristics</th>
<th>Impact on student esteem</th>
<th>Impact on student learning</th>
<th>Student perceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Judgmental/evaluative (summative)</td>
<td>Teacher determines grades; grades come with or without comments</td>
<td>Students who did well feel good about themselves; students who did not do well wonder whether they’re smart enough</td>
<td>Surface learning is most likely since the level of cognitive demand is low; students memorize formulas, look for “tricks” and focus on the grade, not their understanding</td>
<td>Mistakes are bad; there must be a math gene; I need to do more memorizing; there are so many tricks to memorize</td>
</tr>
</tbody>
</table>
**Table 2.1—Continued**

<table>
<thead>
<tr>
<th>Feedback type</th>
<th>Characteristics</th>
<th>Impact on student esteem</th>
<th>Impact on student learning</th>
<th>Student perceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Descriptive/informational (formative)</td>
<td>Teacher responds on the basis of student goals; feedback indicates what needs to be done to reach the goals; praise for what students did well; suggestions for how to improve; praise for trying so hard</td>
<td>All students feel their efforts are recognized, feel they can succeed if they work hard, and “buy in” to the assigned tasks</td>
<td>Deep learning is likely to occur since the level of cognitive demand is high; students advance toward stated learning goals; students display high-quality learning aimed at understanding and improvement</td>
<td>Effort is the key to success; mistakes identify areas that need more effort; learning is fun</td>
</tr>
</tbody>
</table>

**Motivational feedback**

Motivational feedback is designed to make learners feel that their work is recognized and that they are making progress, as well as to encourage and support learners. Motivational feedback is not designed to give guidance on how to improve the learner’s reasoning or to move students forward in the learning process; you can give such guidance on summative assessments.

**Evaluative feedback**

Evaluative feedback is designed to measure student achievement with a score or a grade and to summarize student achievement—a summative assessment. Evaluative feedback is not designed to give guidance on how to improve the learner’s reasoning or to be given on formative assessments, since it is not designed to move students forward in their understanding.

**Descriptive feedback**

Descriptive feedback is designed to be the following (Wiggins 2005):

- Feedback for learning, a formative assessment
- About the work, not about the student
- Neutral, with nothing in body language, facial expressions, or verbal or written comments to suggest that the student work is erroneous
- Timely
- User-friendly in approach and amount
- Descriptive and specific in regard to performance
- Consistent
- Expert
- Accurate
- Honest, yet constructive
- Derived from concrete standards
- Ongoing

Descriptive feedback is not designed to be a summation of learning; to be about the student, only about the work; or to be judgmental.

**Effective feedback**

Effective feedback is designed to do the following:

- Move students forward in their understanding
- Support students to internalize the feedback and to use the suggested strategies independently on future work
- Be used by learners to independently move their reasoning to the next level
- Use criterion-based phrases to describe the strengths and weaknesses of learners’ work
- Limit feedback to one or two traits or aspects of quality at a time
- Offer an opportunity to “redo” work according to the effective feedback
- Encourage self-reflection

One of the most effective practices for improving student proficiency is to engage students in self-reflection, but students need guidance in this process. Students who regularly receive descriptive or effective feedback are more likely to reflect on their learning progression than are those receiving just grades or a combination of grades with motivational feedback.

**Using assessment to plan instruction: Exit cards (exit slips)**

How do you know what your students learned in any given mathematics class? Do you know for sure whether your students learned what you think they did? One method of determining whether your students learned what you planned is to use exit cards. Exit cards are a quick (no more than five minutes) formative assessment strategy for measuring student learning. Reserve the last few minutes of class time to give students an index card on which they briefly write about what they learned in the lesson or answer one question or perform one computation that you determined beforehand to be a gauge of what you think they should have learned. On the way out the door, students deposit their exit slip into a labeled receptacle.

One teacher uses a general “3–2–1” exit card, and students use index cards to record their responses (fig. 2.10). This teacher begins the next day’s lesson by addressing the questions the students had about the prior day’s lesson. Sometimes she poses questions for the rest of the class to discuss; other times, she simply raises the question and supplies an answer. This teacher has found that when her students realize that they have to complete the exit card each day, they tend to pay closer attention to the mathematics they are investigating and the problems they are solving. If she forgets to hand out the exit cards, the students remind her. Asked why they would prompt their teacher to ensure that they filled out the exit cards, some students replied, “If I think
and write about what we did in class, it makes it easier to remember and helps me tell my parents what we did in math class.” Other students shared that “It makes me realize how hard I worked in math class.” Still others responded, “It helps me sort out my thoughts before I have to start thinking about another subject.”

These general exit cards can yield information about individual students if you ask them to put their names on the cards. But because the questions are so open, they will not give you evidence that all students have acquired the specific knowledge, understanding, and skill in the mathematics that you had planned for them.

A second teacher uses the exit cards to stay informed about how well her students are maintaining their computational skills. Most of her exit cards are content based. She discovered that her eighth-grade students did not take the work seriously, so she instituted an evaluation system for the cards. Although this grading system is used as a summative assessment, the teacher does
use the information to drive her instructional decisions. Together with her students, she developed a rubric for the exit cards (fig. 2.11).

<table>
<thead>
<tr>
<th>2 points</th>
<th>Correct computation with supporting work</th>
</tr>
</thead>
</table>
| 1 point  | Correct computation lacking supporting work  
|          | OR  
|          | Incorrect computation but evidence of a sincere attempt to answer the question |
| 0 points | Incorrect computation without supporting work  
|          | OR  
|          | No work at all |

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**Fig. 2.11. Grading rubric for students’ exit cards**

At the end of the week, this teacher adds the total points and uses the grade as a quiz grade. She reports that her students now take the exit card activity seriously. The exit cards add another piece of evidence about her students’ proficiencies, and parents are happy to see their children still doing some math to which the parents can relate. But most important, this teacher still uses the student responses to help her prepare the next day’s lesson.

A grade 8 student whose teacher asked the students to record the “muddiest point” in the day’s lesson completed the following exit card (fig. 2.12):

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**Fig. 2.12. An exit card showing material unclear to a student**
On the basis of the exit card responses, this teacher planned time for his students to do more problems that incorporated surface area of various figures.

Another teacher asked her students to list the problem with which they had the most difficulty. Many students struggled to determine which was larger: \(\frac{1}{(-9)^8}\) or \(\frac{1}{8^9}\) (fig. 2.13).

**Fig. 2.13. Exit-card responses to a which-is-larger problem**

Before instituting exit cards, this teacher would have moved on to the next lesson in her text. After seeing her students’ confusion on the exit cards, she decided to spend at least another lesson determining strategies for her students to make comparisons with exponents.

Another teacher asks her students to list what they learned in mathematics class. Figure 2.14 illustrates how differently students in one class can respond.
Fig. 2.14. Exit-card responses describing what students learned

Notice the misconception in figure 2.14A. The student believes that finding an equivalent fraction actually changes the value of the original fraction. The student appears to believe that the larger the numbers in a fraction, the greater the value. The work in figure 2.14B illustrates a learned procedure for finding equivalent fractions. This exit card clearly indicates that the student can calculate equivalent fractions procedurally, but exactly what conceptual understanding the student has is unclear. The language “how to make small fractions into out of 100 fractions” lends itself to having an interview with the student.

After examining the exit cards, this teacher decides to assign more problems that include finding equivalent fractions as part of the solution process.

Including exit cards in a lesson benefits both the teacher and the student. Although traditionally exit cards were considered summative assessment, with teachers using them for quiz grades, many teachers use them formatively to plan the next day’s lesson. The beauty of using exit cards is that one day’s lesson depends on how well the students demonstrated understanding on the previous day. Rather than moving on to the next lesson in the textbook, the flow of daily lessons reflects student understanding from one day to the next. The daily inclusion of these formative assessment strategies and protocols will enable you to use the evidence of your students’ learning to give you information for your instructional decisions, help guide your grouping, and assist you in the types of questions you might pose. When the evidence indicates that students do not display a deep level of understanding, it may indicate the need to engage the students in different activities, tasks, and problems.
Selecting tasks or problems for range questions and exit cards is a skill that develops over time. Select range questions to give you a sense of where your students are on the continuum of their learning progression. These tend to be broad questions that tease out how students are thinking, what they recall from past experiences, and how or whether they make connections. Select exit card/slip questions to tease out what your students are bringing away from class that day. These questions are usually more focused than range questions, may relate to procedural skills, and should be selected because of their potential for exposing those understandings or misconceptions that students are walking out of class with.

REFERENCES


