A sk a person on the street what they think of mathematics and you are unlikely to get a lukewarm answer. Some people will happily claim they love it, but more people will shudder in horror or sheepishly admit it is one of their biggest weaknesses. Mathematics conjures up memories of sitting in classrooms memorizing disconnected topics that do not seem to apply to the real world. After all, when does anyone ever use a quadratic formula? When asked about English or reading, few would answer: “I’ve never been very good at English” or “I’m just not a reading person.” But these kinds of statements are made every day about mathematics by a variety of educated people, with no sense that the statements should be questioned. As a society, we perpetuate a myth that there are just those people who are good at mathematics and those who are not. So those who do not see themselves as “math people” do not take it personally. It is just the natural order of things.

But is it?

Researchers who study the brain and the way we process numbers and concepts have shown that we are all hardwired to learn mathematics (Devlin 2001). Even babies as young as six months old can distinguish between small and large quantities (Lipton & Spelke 2003). This is true of every culture, race, and gender. It is not until we enter school that we start to see a fall-off in the number of people who do mathematics well or enjoy doing it. By the fourth grade, we see a real decline in the number of students who understand basic concepts, and this is particularly true for Latino/a and black students. So, if there is nothing wrong with our brains, no genetic predisposition, what is causing this great deficit in mathematical competency? Part of the reason may have to do with how we define mathematics and where we look for it.

This paper focuses on Latino/a and black students in the United States because of the persistent trend in these populations (especially among low-income students) for low performance on standardized tests and their lack of representation in advanced mathematics courses and mathematics-based careers. Of course, many issues make collapsing these two groups into one problematic. For example, many Latinos/as are dealing with issues of language that may not pertain to, or may play out differently for, blacks. That is, mathematics is its own language with terminology that differs from everyday use so that a student who is still learning English will have the challenge of learning two languages in the mathematics classroom. Even among Latinos/as, there are a variety of nations, races, racial phenotypes, and immigration experiences that students bring to the mathematics classroom. The same can be said of blacks, some having more recent ties to specific African, Caribbean, South American, and Asian countries, while others have ancestors who were brought over through slavery. Blacks, too, may have issues of language that racialize them differently or contribute to problems of miscommunication with teachers and others. Furthermore, class status may alter the way students receive cultural cues. There is also the fact that many Latinos/as are also black (e.g., Dominicans, Afro-Brazilians). Clearly, there is not a singular or universal Latino/a or black experience in mathematics. However, we believe the major forms of marginalization that are experienced during school by Latino/a and black youth in the United States are similar.

More so than their white peers, black and Latino/a students are strongly affected by the rigor of the mathematics curriculum, the quality of their teachers, and the beliefs teachers hold of them. For example, a review of studies asking teachers to assess the current or future ability of Latino/a and black students shows a statistically significant bias toward negative stereotypes and low expectations (Baron, Tom, & Cooper 1985). Many teachers believe the black-white achievement gap is at least partially genetic and, therefore, may be sustaining it through a self-fulfilling prophecy. In a survey of 379 secondary mathematics teachers, respondents attributed the
Some researchers have suggested that because Latino/a and black students worry about fulfilling negative stereotypes, they face additional pressure and vulnerability that can lower their performance on standardized tests.

Historically, our understanding of Latino/a and black student achievement in mathematics has tended to focus on comparisons to middle-class white students; today we call this the “achievement gap.” In these comparisons, Latino/a and black students often come up short, reinforcing stereotypes by teachers and others in society about the mathematical capacities of students of color. However, because these studies rely primarily upon one-time responses from teachers and students, they capture neither the history nor the context of learning that have produced such outcomes (Gutiérrez 2008). In addition, most people are unaware that the distributions of achievement for Latino/a, blacks, whites, and Asians largely overlap; in general, there is more variation in achievement within a group than between groups. Perhaps most important, the knowledge captured by standardized tests does not reflect the state of the art about what kinds of mathematical understanding, practice, and disposition are important for students if we expect them to pursue a mathematics-based career, work in an increasingly technological society, or become critical citizens in a democracy.

In fact, the recent release of the Common Core State Standards suggests that, within mathematics, more than just mastering eight key mathematical properties (moving from “novice” to “apprentice” to “expert”), students are also expected to develop a “character” that relates to mathematics (Daro 2011). This notion of building a mathematical identity is something the research community in mathematics education takes very seriously. In fact, teachers are being asked to “empower all students to build a relationship with mathematics that is positive and is grounded in their own cultural roots and history.”
takes very seriously. In fact, teachers are being asked to “start each unit with the variety of thinking and knowledge that students bring to it” (Daro 2011), and to “empower all students to build a relationship with mathematics that is positive and is grounded in their own cultural roots and history” (NCTM 2008).

If scores on standardized achievement tests capture only a fraction of the issues we think are important in mathematical learning, where else should we turn our attention? Perspectives on mathematics as a social activity and how people learn outside of school offer a unique starting point for rethinking the problem of mathematical achievement for all students, and for Latino/a and black students in particular. This focus on student-centered learning can inform different ways of teaching and organizing schooling so that more Latino/a and black students are engaged and learn.

Focusing on mathematics that students learn outside of school (where it happens more naturally) and on mathematics applied to personal/community issues that students bring to school may better highlight the competencies and needs of these students as learners. Starting with mathematics as a social activity (as opposed to a set of skills that schools need to impart on students) may also better connect with the kinds of interdisciplinary learning that individuals will face in life.

This paper examines four categories of research:

> Ethnomathematics (e.g., cultural practices seen as unique to a particular group);
> Adults and others learning to use mathematics (e.g., for professional development in their careers; as part of their everyday practices);
> Students learning in afterschool contexts; and
> Social justice mathematics (e.g., math as a tool for addressing injustices).

The purpose of this literature review is to broaden popular conceptualizations of mathematics achievement of Latino/a and black students. By doing so, it aims to inform and inspire mathematics practitioners to craft innovative pedagogies to better support Latino/a and black youth.

We examine over eighty empirical and conceptual papers from these four categories. We privilege empirical works, distinguishing between studies that document culturally embedded uses of mathematical thinking (e.g., Kenyans crafting baskets with geometric designs) and studies that document students engaged in mathematics for a variety of purposes (e.g., in work settings or for home economics) and how engagement in these mathematical practices bears upon student achievement in school mathematics. In areas where empirical works are scarce, we draw from relevant conceptual papers to make recommendations for future research. Much of the literature comes from journal articles. However, we also considered evaluation reports and reviews of the field commissioned by foundations. This was especially true regarding afterschool learning.

We begin with early studies documenting the varied forms of mathematics that are developed and used by other cultures in their daily practices. From there, we demonstrate that “ordinary” people in the United States (e.g., carpenters, tailors, grocery shoppers) have mathematical abilities and strategies that do not transfer readily to school mathematics, in part because they are not explicitly conscious of the mathematical nature of their everyday tasks. Then we discuss the nature of mathematical learning in afterschool clubs, where novice and expert are blurred and the notion of “playing” with mathematics is more prevalent. Finally, we examine contemporary research showing that students are more engaged in using mathematics as a tool for analyzing problems when the context is injustices in society. We contrast all of this learning with the formal mathematics of schooling and suggest ways to bring the two into better collaboration.

This paper examines a wide variety of mathematical practices and competencies that are missed by school mathematics. Furthermore, it highlights the voices of learners themselves—what meanings they place on mathematics and mathematical learning.
In conducting this review, a number of questions arose:

> **What causes people to do mathematics outside of school hours?** What does that mathematics look like?

> **Are people more competent in mathematics outside of school?** If so, why?

> **What keeps students from being able to apply what they know outside of school to the mathematics classroom?**

> **Why is it that an afterschool mathematics program that takes place at a school can look so different from a mathematics classroom in that very school?**

> **What kinds of social justice issues are on the forefront of students’ minds?**

> **Can topics in a high school mathematics curriculum (e.g., calculus) be used to explore social justice issues?**
Philosophers, sociologists, and anthropologists who study mathematics have long argued that “school mathematics” is but one small version of the many forms of mathematics practiced in the world. Moreover, they have made convincing arguments that mathematics does not operate outside of individuals, morals, or power relations (Brown 1994; Clarke 2001; Ernest 1994, 2004; Fitzsimons 2002; Restivo 1994, 2007). Even mathematicians, when asked, “What is mathematics?” offer a whole host of definitions, including definitions recognizing that humans create mathematics (Burton & Morgan 2000). One needs only consider how contemporary mathematics as a research field is constantly changing and allowing for internal contradictions (e.g., catastrophe and chaos theory; undecidability; uncertainty; fuzzy logic) to see that mathematics is neither a static entity nor a field where those who practice it seek to obtain one right answer (Kline 1980). Yet, in school, we talk about mathematics in ways that ignore the fact that humans create multiple mathematics; that mathematics has a history; and that people across the globe practice it in many different ways. In some countries, mathematics is not talked about in the singular form (math) as it is in the United States; for example, it is referred to in Great Britain as “maths,” even in everyday speech. It may be that our language for talking about mathematics in the United States further engrains in students and teachers the idea that mathematics is a single entity.

Over the past two decades, research in mathematics education has moved from an emphasis on cognitive psychology (mathematics as something that happens in the minds of individuals) to mathematics in social interactions (Lerman 2000). For example, we now see knowledge as intricately tied to a person’s context, including why and with whom one is doing mathematics. From the point of view of mathematics as a social activity, teachers need to recognize it is not productive to think of their work as “teaching” students to think mathematically. Rather, teachers initiate students into mathematical communities and practices. As Sal Restivo (2007) writes:

Mathematics students might learn more effectively by recapitulating the ways the mathematical community came to collectively grasp concepts and ideas. . . . I would certainly advocate teaching mathematics in the context of their historical development. The historical, social, and cultural contexts cannot be separated from the substance of mathematical objects, concepts, and ideas.

This social perspective is so prevalent that when talking about what students “know,” many mathematics education researchers do not just consider whether students have mastered a set of predetermined procedures or facts; they also place great emphasis on identity—whether students think of themselves as people who do mathematics and how students position themselves with respect to each other in the mathematics classroom (Cobb et al. 2009; Martin 2006a, b; Esmonde & Osuna-Langer forthcoming). Given the research on teachers’ beliefs and stereotypes, issues of building strong and positive identities for Latinos/as and blacks are especially important to consider in teaching and learning.

Related Paper in the Students at the Center Series


If we consider mathematics as a social activity, where might that lead us? How does looking beyond the school walls help us better understand what mathematics students know and are doing? How can this understanding of learning outside of school help us better support Latino/a and black students in their mathematics learning in general?
O

ver the past three decades, a variety of research fields that have developed within mathematics education speak to the question of how people learn mathematics outside of the institution of schooling. We review here four main research areas: ethnomathematics; learning mathematics out of school and adults learning mathematics (grouped as one section because these literatures overlap to a great extent); afterschool mathematics programs; and social justice mathematics. Although similar, each of these approaches operates with assumptions and goals that have left them largely disconnected. By combining these different fields and drawing out key features, we offer a more comprehensive vision of what student-centered learning could be.

ETHNOMATHEMATICS

Anthropologists who study mathematics have documented that not only do all people do mathematics, but a variety of forms are practiced in different cultures. In fact, many believe that humans developed mathematics in order to describe the world around us and help us solve everyday problems. Viewing mathematics as a tool to describe the natural environment explains how very different people on different parts of the globe throughout history could create a fairly universal mathematics. And yet differences between cultures may have led to different forms being practiced. Researchers in ethnomathematics argue that the kinds of mathematics developed are partially influenced by the peoples who create them (D’Ambrosio 2006). Having developed within countries that were once colonized and that today oppose importing Western curricula, one of the primary goals within ethnomathematics is to highlight the contributions of different, mainly non-Western cultures to the field of mathematics.

At one level, this work includes documenting the mathematics that have developed throughout time (e.g., in ancient Egypt, Babylonia, India, China, and the Arab world) (Joseph 2010). However, this work also shows that indigenous peoples and adults with diverse perspectives on the world develop diverse mathematical practices (Barton 1996; Bishop 1988; Knijnik 2007; Rambane & Mashige 2007). For example, researchers studying number and pattern in South African cultures have highlighted the roles women play in reproducing geometrical patterns and tessellations through the weaving of baskets and cloth (Gerdes 1997). A common approach is for an anthropologist with extensive knowledge of mathematics to spend large amounts of time within a given population, learning how to do the mathematical work that local people do. In this sense, “ethno” refers to an identifiable cultural group (not a race or ethnicity) (D’Ambrosio 1985; 2006), such that even professional mathematicians could be seen as producing a form of ethnomathematics (Borba 1990; Powell & Frankenstein 1997).

Much of the ethnomathematics research that is readily available highlights the games that are played in many African cultures and how those games draw upon mathematical principles familiar to Western mathematics (Zaslavsky 1998; Crane 1982). One example is the set of games known as *mancala*, where stones are evenly distributed into separate cavities of a long board with two rows (resembling the base of an egg crate holding a dozen eggs). Two players alternate picking up the rocks in one cavity and distributing them equally in the successive cavities until no more rocks are left in the players’ hands. If the last rock lands in a cavity where there are other rocks, the player can continue to play with those rocks until s/he lands in a space with no rocks or home base. The goal is to start picking up rocks so as to end up with the last rock either in a cavity with a large number of other rocks or in one’s home base. Playing...
the game well requires the player to plan ahead where s/he wants the last rock to land and to also consider, among all of the permutations that could be taken, which starting point will result in the longest play and most rocks in home base.

In looking across cultures, researchers have classified the general forms of mathematics that are practiced by all humans. The most cited classification system, developed by Alan Bishop (1988), argues for six basic categories:

> **Counting** is “the use of a systematic way to compare and order discrete phenomena. It may involve tallying, or using objects or string to record, or special number words or names.”

> **Locating** is “exploring one’s spatial environment and conceptualizing and symbolizing that environment with models, diagrams, drawings, words, or other means.”

> **Measuring** is “quantifying qualities for the purpose of comparison and ordering, using objects or tokens as measuring devices with associated units or ‘measure-words.’”

> **Designing** is “creating a shape or design for an object or for any part of one’s spatial environment. It may involve making the object, as a ‘mental template,’ or symbolizing it in some conventionalized way.”

> **Playing** is “devising, and engaging in, games and pastimes, with more or less formalized rules that all players abide by.”

> **Explaining** is “finding ways to account for the existence of phenomena, be they religious, animistic, or scientific.”

Bishop’s work has a unifying sense and reminds us that at some level, mathematics is practiced in the same way across the globe.

Beyond classifying the forms of mathematics as they are practiced, ethnomathematicians also document that what the West often takes to be the exclusive knowledge of professionally trained mathematicians exists throughout the world. Among other things, researchers have shown that in the Marshall Archipelago, where sailing is integral to life and wave piloting is essential, the use of stick charts (maps) relies upon unique geometric and algebraic renderings of the oceans. The intricate designs (kolam) made of rice flour that are created on the threshold of a household by Tamil Nadu women of India represent transformation and superimposition of basic subunits (similar to but different from fractals). And abstract calendars used by the Maya and Balinese cultures show that not all people think of time as a linear progression in static units or as tied to the sun, moon, or other physical object (Ascher 2002). Similarly, the Xavante peoples of the Brazilian Amazon use a binary system in which units are not individual but paired (one-one or one-many), thereby challenging the deeply engrained Western belief that $1 + 1 = 2$ (Ferreira 2001).

Few of the peoples documented in these studies have had formal schooling. Rather, they have developed these ways of using mathematics through learning from others in their community. Studies of ethnomathematics illustrate that not only do other cultures practice mathematics in sophisticated manners, but also that mathematics takes on different forms in different places. **There is not one mathematics that is found everywhere in the world.**

In reading this work, it may feel like ethnomathematics deals with something in the past, with primitive cultures that do not interact with the mainstream. However, that does not seem to be the case. While early studies of ethnomathematics focused on the variety of mathematical practices of diverse peoples, contemporary studies seek to highlight the asymmetrical power relations that arise when different mathematical practices are developed and maintained. For example, a recent study focused on the way a group of landless peasants in Brazil have fought to maintain the effective system they developed for measuring land plots before the school testing industry and government officials began requiring an official European system (Knijnik 2008, 2011). Testimonies by the peasants indicate that, on the one hand, doing mathematics in school denies them knowledge they have developed outside of school and are accustomed to using. It is not that they are incapable of learning the new system, but their sense of calculating the areas of land with their own system feels more connected to their roots. In this sense, it is part and parcel of their way of being and reflects the recent stance on equity taken by the National Council of Teachers of Mathematics, arguing
that mathematics should be grounded in students’ cultural roots and history (Borba 1990; NCTM 2008). On the other hand, the landless peasants’ testimonies also reflect the belief that the mathematics education they have received has not given them enough formalism and abstraction to help them negotiate a language that has traditionally kept them as outsiders.

Recent studies focusing on the perspectives of learners seem to point to the importance of learners having reference items for doing mathematics. A study of women 14 years and older in the suburbs of Brazil indicates that being able to work with familiar contexts (e.g., beans, rice, sugar) makes doing school mathematics problems easier (Fantinato 2008). Similarly, Knijnik’s landless peasants reported that concrete materials have made it easier to learn school mathematics and to teach it to others.

How does this relate to Latinos/as and blacks in the United States? For the most part, mathematics curricula rarely teach the history of mathematics—how it was developed by different peoples in different parts of the world or how it is still developing. Any history that is conveyed to students tends to be in the form of textbooks crediting mathematical theorems and discoveries to individuals of European descent (e.g., Newton, Euclid, Pythagoras, Euler, Gauss, Déscartes, Fibonacci). Few students realize that the Pythagorean theorem was known by the Babylonians and Chinese more than a millennium before Pythagoras lived or that the numeral system we use today is Hindu-Arabic. Omitting this dynamic history from the classroom can give students the impression that excellence in mathematics is the exclusive domain of Europeans.

In contrast, researchers who study ethnomathematics have suggested how educators might incorporate it into school (Presmeg 1998). Some have argued that when students are connected to the things they are learning about (by introducing topics based upon cultural experiences and ideas they already have), they will be more motivated to learn (Begg 2001). This line of thinking follows a commonly held view in mathematics called “constructivism” which suggests that learning is best facilitated when new knowledge can be scaffolded onto previous knowledge; this is especially true for English learners who are in the process of learning mathematics (Gutiérrez 2002a, forthcoming; Moschkovich 2002). One design experiment showed a positive correlation between using a software tool that models tiling patterns to engage and support 18 Indian third graders in learning fractions (Sankaran 2009). The approach they use seems plausible with older students as well.

Still, others worry that a purely ethnomathematics approach (based upon the experiences that students bring with them) can turn schools into labor training institutions and possibly reinforce the subordinate position of marginalized students (Rowlands & Carson 2002). These researchers argue that problem posing and problem solving require an abstract understanding of mathematics that is not present if the starting point is always students’ experiences.

Although ethnomathematics can highlight the contributions of non-Western cultures to mathematics and the unequal power relations that arise when schools ask cultures to ignore cultural practices they have developed, a number of challenges arise for teachers interested in applying an ethnomathematics approach to their classrooms. First, in order to maintain a rigorous mathematics classroom, teachers must have a broad understanding of the history of mathematics, the identities of their students, and how the two might interrelate.

Second, there is a disjuncture between the mathematical practices that have been documented around the world and ways of relating these practices to teaching/learning. That is, few sources of lesson
plans or educational activities are available for teachers to use. This is particularly true at the middle and high school levels. We found two exceptions: activities created by the Exploratorium Museum in San Francisco; and Culturally Situated Design Tools created by Ron Eglash.

**THE EXPLORATORIUM**

One set of ethnomathematics activities available for teachers was generated and piloted by the Exploratorium (Bazin, Tamez, & Exploratorium Teacher Institute 2002). Its 14 inquiry-based activities begin with historical background on the context in which the mathematics occurs and make suggestions for how teachers can launch and assess each activity. The suggestions include such topics as: ancient Egyptian numeration, the *quipus* numerical system of the Inca, a game of solitaire from Madagascar, Mayan numeration and calendars, African *sona* (sand) drawings, and the basket-weaving patterns of many cultures. For example, the activity of *sona* drawings introduces students to the fact that in the southwest African region of the Chowke, people tell stories while drawing lines in the sand that weave in and around dots that are arranged in a rectangular array. The storyteller draws these lines without stopping, obeying rules of the manner in which the lines can weave in and out. Students are encouraged to explore how knowing the rectangular array (e.g., four by six) can predict how many closed lines (two in this case) are required to make the drawing. At an abstract level, this work involves calculating the greatest common divisor. We were unable to locate any student assessments for teachers using these activities.

Beyond having well-developed activities, incorporating ethnomathematics into school also requires that teachers know their students well and can consider their students’ roots or previous experiences in an effort to link this previous knowledge with the abstract knowledge that schools require (Miranda 2008). That is, without thinking carefully about how to use activities like the *sona* drawings, teachers may inadvertently convey to their students that blacks are primitive or do not make modern-day contributions.

**CULTURALLY SITUATED DESIGN TOOLS**

Focusing on African fractals and design principles in modern culture, Eglash has developed a set of Culturally Situated Design Tools that target Latino/a, black, and Native-American students (Eglash 1999, 2010; Eglash et al. 2010). His design tools enable students to reproduce art by leveraging underlying mathematical principles in such things as: Latino-Caribbean percussion and hip-hop rhythms (ratios); graffiti (Cartesian and polar coordinates); corn-row hairstyling (transformational geometry, fractals); break dancing (rotational and sine function); and pre-Columbian architecture (symmetry, pre-algebra). Students learn about the cultural backgrounds of the art being modeled and get tutorials on how the software works before being encouraged to invent their own designs. Teachers get lesson plans, evaluation materials, suggestions for how to use the design tools, and connections to the standards of the National Council of Teachers of Mathematics.

The complexity of the designs challenges a primitive, static, or overly exotic view of culture and highlights the fact that many cultural artifacts show mathematical principles that are intentional, rather than due to individuals who are mindlessly copying others in their community. Unlike the counting systems and calendars that directly translate to Western mathematics, many urban and modern cultural practices have mathematics embedded in their processes (e.g., iteration in bead work; Eulerian paths in sand drawings). As such, they require students to create mathematical models of cultural phenomena.

Eglash has studied the impact of his culturally situated design tools on Latino/a, black, and Native-American secondary students, some of whom their teachers described as “problem students.” He found that students feel a sense of agency in creating their own designs. They also greatly improve their attitudes toward mathematics and connect social and technical domains in the creation of their identities. When given the choice to invent new designs, most students appropriate the software tools to express their identities. For example, several Puertoriqueño/a students have used the iteration program for bead work to create a Puerto Rican flag. Several low-
income black students have used the iteration program to write their initials in a way that is similar to graffiti tags. And Latino/a students have explored hip-hop music to find the least common multiple between the rotations of the rhythm wheels. In a survey of 175 randomly selected low-income eighth-grade students who have used the tools extensively, Eglash found that Latino/a and black students showed a statistically significant increase in their interest in information technology and computer-related careers; he did not find any increase among European descendant students. Moreover, three high school teachers using the tools conducted studies of their students’ learning (one or two classes per teacher) and found that students showed a statistically significant increase in grades and in pre- and post-tests in pre-algebra concepts.

SUMMARY

What we learn from these studies is that the forms of mathematics we privilege in school (e.g., Euclidian geometry; Cartesian coordinates; the base-10 counting system) are not the only mathematics that people use. And no single mathematics is produced. Moreover, the mathematical practices that have developed among different cultures serve a purpose. That is, people use mathematics not just to display knowledge to others (get good grades) as happens in school, but to accomplish something in everyday life. We also see that people learn mathematics not necessarily from someone called a teacher but also from someone in their environment who has apprenticed them into this way of using mathematics. Furthermore, individuals use mathematics in the particular ways they have learned because they make sense.

At a basic level, this research raises several questions for student-centered learning. For example, how might learning the history of mathematics and the different ways in which cultures across the world use mathematics interest students in learning more about the subject or making comparisons among its different forms? Might an emphasis on the history of mathematics (how it was created in different places at the same points in history) help them see that everyone does mathematics (including Latino/as and blacks), not just the Greeks, and that everyone changes their mathematical practices over time?

How might that affect the development of students’ mathematical identities?
How might school look if students (especially immigrants) were encouraged to use forms of mathematics they knew from their home countries?
How might students feel about themselves and their ancestors if the mathematics they knew from practices outside of school were valued or built upon? How might issues of who is the expert or novice change if community members who knew of mathematical games, weaving forms, or other practices were invited into the school to share their work and to help others learn to do it?

LEARNING MATHEMATICS: OUT OF SCHOOL AND ADULTS

What do studies conducted in North America or practices that are common there tell us about how people learn mathematics? A better understanding of the relation between mathematical use, reasoning, and motivation may be key.

Consider the typical mathematics classroom. A common question posed by students is: “When am I ever going to use this?” This question is especially pertinent for students who have not fared well in school, including Latino/a and black youth. Some students may feel that school requires them to park their identities at the door. Others may simply question a disconnected way of knowing. Regardless, the typical argument made by schools and teachers for why individuals need to learn mathematics is that the knowledge they gain is general enough to transfer to their everyday lives. Some might even say that learning mathematics helps students become critical thinkers. Teachers and textbook publishers seem to be comforted by the fact that they are creating “real-world” problems. Yet studies of people using mathematics in their work and in their everyday lives seem to challenge claims that mathematical thinking taught in schools can be applied to life or that the problems used in textbooks and mathematics lessons reflect the real world (Frankenstein 2009; Dapueto & Parenti 1999).
When adults who return to school to learn mathematics are asked why they do so, the answer is not that they seek to develop more abstract or generalized ways of using mathematics so they can apply these to their everyday working contexts. Instead, they report wanting to help their children with their homework (in ways schools expect children to represent their knowledge) or to prove to others they are smart (because mastering mathematics implies a kind of intelligence) (Wedge 2010).

Studies of people using mathematics outside of school seem to illustrate that individuals do not apply rules or ideas they have learned in the mathematics classroom to real-life problems; rather, they draw heavily on a familiar context in which they participate (e.g., Smith 2002). Poor children selling candy and melons on the streets of Brazil can calculate complex sums in the context of their work but not in similar paper-and-pencil, “school like” problems (Nunes, Schliemann, & Carraher 1993). The same has been found for adult carpet layers, grocery shoppers, interior designers, retailers, and restaurant managers (Millroy 1992; Saxe 1998, 1991; Lave 1988; Lave & Wenger 1991; Masingila 1994; Carraher et al. 1985; Schliemann 1985). For many of these individuals, the “naked” (stripped of context) problems that researchers presented to them as equivalents to what they were doing in everyday practice were seen as not equivalent at all, leading to nonsensical solutions (Carraher & Schliemann 2002). Estimating sums was easier for fourth-grade students in Italy when they could use grocery receipts because they could reason about the appropriateness of item prices with which they were familiar (Bonotto 2001). Moreover, these students reportedly could make better inferences and check the reasonableness of their answers because of the familiar context.

Looking across a variety of studies, some researchers have sought to categorize more broadly how school mathematics learning differs from mathematics learning outside of school and have identified four significant differences (Resnick 1987):

### LEARNING MATHEMATICS IN SCHOOL VS. OUTSIDE OF SCHOOL

<table>
<thead>
<tr>
<th>In School</th>
<th>Out of School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual thinking</td>
<td>Shared thinking</td>
</tr>
<tr>
<td>Pure thought</td>
<td>Using tools</td>
</tr>
<tr>
<td>Manipulating symbols</td>
<td>Contextualized reasoning</td>
</tr>
<tr>
<td>Generalized learning</td>
<td>Situation-specific competencies</td>
</tr>
</tbody>
</table>

The distinctions in the kinds of learning are important: they point to why school mathematics does not always make sense to students or serve as a means for feeling competent. Consider this example of a mathematical exercise on “combining like terms” that students might see in an algebra course: $2x + 3y = ?$. It is not difficult to see how a student can make errors (e.g., combining unlike terms when working with variables) if manipulating algebraic symbols never involves thinking about what those symbols refer to (how each variable represents a different entity that prevents it from combining with another):

<table>
<thead>
<tr>
<th>In School</th>
<th>Out of School</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x + 2y = 5xy$</td>
<td>5 apples + 3 bananas = 5 apple-bananas</td>
</tr>
<tr>
<td>Could seem to make sense</td>
<td>No such thing as apple-bananas</td>
</tr>
<tr>
<td><strong>Errors persist</strong></td>
<td><strong>Reality tells us this is incorrect</strong></td>
</tr>
</tbody>
</table>
Although significant reform efforts in mathematics education have been underway since these studies were reviewed in 1987, the four differences persist. Even highly technical professionals like radiologists reading x-rays or nurses calculating drug dosages use processes different from those taught in medical schools, through textbooks, or on medical rounds (Lesgold et al. 1988; Hoyles, Noss, & Pozzi 2001). Their mathematical reasoning tends to be grounded in the contexts in which they are working and in relation to others with whom they work. Other researchers have made similar claims, suggesting that the main differences between out-of-school and in-school learning are that in the former, problems are embedded in real contexts that are meaningful and therefore provide the motivation for learners to want to solve, while in the latter, the thinking processes used by learners are different from and arguably of a higher level than those taught in schools (Masingila 2002).

Studies of mathematical learning in out-of-school contexts also highlight the importance of apprenticeship (Lave 1988; Lave & Wenger 1991; Masingila 1994). Teaching in out-of-school contexts is not explicit; it is observed. Here, signs of learning are in the form not of individual acquisition of knowledge but of greater participation (performance) in the practice.

A small number of studies have looked at adolescents practicing mathematics outside of school (Masingila 2002; Nasir 2000, 2002; Nasir, Hand, & Taylor 2008; Nasir & de Royston forthcoming). Observations of black middle and high school basketball players indicate that they are competent at calculating averages and percentages for the free-throw shots of a given player when the context of the problem is a basketball game rather than a school mathematics worksheet. Furthermore, like the findings in ethnomathematics, the kinds of strategies used in the basketball context differ. For the school mathematics problem, players tend to incorrectly remember or misapply algorithms such that their strategies for finding an answer are reduced to mere manipulation of the symbols.

It is not just the strategies and understandings of mathematics that differ across the in-school and out-of-school tasks presented to the black youth; it is the differences in their sense of themselves—what they are capable of within mathematics in the different settings—that is important. This focus on identity is key for relating to the state of the field in mathematics education and for creating “character” around mathematics that is called for by the Common Core State Standards. For example, when players are asked to solve the problems in basketball contexts first, they score better on both types of problems. However, when they must solve the problems first in abstract school terms, they score lower on all of the problems. The researchers surmise that failure to solve the problems in the school context makes it difficult for players to call up more complex reasoning strategies with which they are familiar. When asked to solve the problems first in the basketball context, they seem to possess the confidence to persist in the school-based problems, even if their understanding of the algorithms is weak.

Few of the studies we located asked students for their views on learning mathematics outside of school. One exception is a study that asked 20 middle school students (10 urban, 10 suburban) to record their uses of mathematics outside of school hours in a log (Masingila 2002). Using Bishop's six categories for analyzing their responses, this study found that students who have broadened views of what mathematics is (beyond counting, measuring, and designing) provide a greater number of examples of mathematics and include all six categories (including locating, playing, and explaining).

As such, merely asking students to take note of mathematics outside of school may not be enough to broaden their views of what mathematics is or how they practice it. A case study of adults in a folk school and prison in Finland highlights the fact that people tend to think mathematics primarily has to do with computational skills and do not see themselves as good at mathematics (Hassi, Hannula, & Nevado 2010). Similar theoretical arguments suggest that most people have very limited views of what mathematics is (Klinger 2011). The adults in these studies report that word problems aiming to model real-world situations (e.g., home, workplace, commerce) are not meaningful. One study
It seems reasonable that if adolescents are encouraged to model phenomena with which they are familiar and are expected to look for generalizations in the data they are generating, they may be more likely to learn the abstract, formal mathematics that is required in school.

Moses recognizes that in getting students to transition between arithmetic and algebra, they must not only be able to count (number); they must also consider direction (positive versus negative numbers). Without a clear understanding of both dimensions, algebra can be confusing. As such, Moses’s Algebra Project approaches this issue by having students move through a five-step process that chronicles an event: physical event; the picture or model of the event; intuitive (idiomatic) language description of the event; a description of the event in regimented English; and symbolic representation of the event. Students do this by taking a subway trip and then mapping out their route, answering questions about “how many” and “which way.” Adolescents use this process to model phenomena from their everyday lives (e.g., cooking, painting, repairing). A key feature of the Algebra Project is beginning with where the students are and the experiences they share. Then students reflect on those experiences, draw conceptual connections to them, and finally apply that to their conceptual work:

Students learn that math is the creation of people—people working together and depending on one another. Interaction, cooperation, and group communication, therefore, are key components to this process. . . . Cooperation and participation in group activities, as well as personal responsibility for individual work, become important not only for the successful functioning of the learning group, but for the generation of instructional materials and various representations of data as well (Moses & Cobb 2001).
Evaluations of the Algebra Project indicate some success with this approach (NRC 2004). Anecdotally, the first group of students who graduated from the project enrolled in high school in geometry, and many have gone on to medical and other graduate schools. In Arkansas, 7 out of the 11 cohorts of students that were followed longitudinally showed at least a 10-point increase in mean-scaled scores on the SAT-9 a year after being in the program. Moreover, students scored at or above the proficiency level in all of the Arkansas sites, as compared with controls who declined or stayed at their proficiency levels.

Teachers who receive professional development from the Algebra Project are asked to reflect upon and address community problems. Much of the work of the Algebra Project relies upon older people who are in constant contact with a small group of youth with whom they develop meaningful relationships (Moses et al. 1989). It is unclear whether such relationships can be “scaled up.” In the words of Moses and Charles Cobb (2001):

In the Algebra Project we have found that teachers, like students, also need nonthreatening arenas where their concerns can be articulated. . . [T]he question remains as to whether something with that level of comfort can be institutionalized and become integral parts of school systems.

Even if scaling up might be difficult, a number of promising practices in the Algebra Project should be incorporated into more learning environments for black and Latino/a adolescents.

Some research suggests that better connecting of out-of-school and in-school practices and learning can help students:

> Prepare to deal with novel problems (both real world and non-real world); and

> Acquire the concepts and skills that are useful to solve routine everyday problems (both real world and non-real world) (Masingila 2002).

Other studies of Latino/a parents learning to use mathematics suggest that building upon students’ previous cultural experiences—what some researchers have termed “funds of knowledge”—can help address issues of equity in schools. One model is for teachers to go into the community and observe and interview families about the kinds of activities (e.g., chores) students do at home. They can build upon these forms of expertise in the classroom. However, a “funds of knowledge” approach to teaching is not simple (Civil 2002, 2007; Gonzalez et al. 2001; Moll et al. 1992). It can lead to stereotypes about particular cultural groups (e.g., presuming what kinds of experiences Latino/a adolescents bring to school) or require copious amounts of time getting to know students and their communities.

Like those who have studied ethnomathematics and shown that differences in mathematical practices create power dynamics, some researchers who study multiethnic classrooms have found that schools often ignore or even reject the knowledge that students possess from their experiences outside of school (Abreu 1999; Abreu & Cline; 2005; 2007; Abreu, Cline, & Shamsi 2002, 2000 as reported in Abreu & Cline 2007; Adler 1999; Setati et al. 2002; Setati & Moschovich forthcoming). Studying farmers and school children in a sugar cane farming community in Brazil, researchers report (Abreu & Cline 2007):

> “When farmers were exposed to modern institutions (schooling, technological innovation), this raised their awareness that some forms of knowledge were perceived as more ‘powerful’ than others.

> “Farmers passed on traditional knowledge to new generations in a selective way so that it was more likely to be passed to a child who failed at school than to a successful one.

> “Farmers’ mathematical knowledge was denied the status of ‘real’ knowledge even by children engaging in their family’s practices.

> “Farmers valued schooling and let their children attend for several years even when they failed to progress and learn.

> “School mathematics could be openly brought into farming, so that young people generated hybrid strategies and showed an understanding of how to convert between systems (e.g. of the equivalence between farming measurement and formal metrics). But this relationship was asymmetric (i.e., farming mathematics was not accepted at school).”
Follow-up questions with teachers of these students indicated that teachers presumed students would not want to become farmers. Therefore, they had little incentive to bridge the out-of-school knowledge and the in-school knowledge. Further research that this team conducted in multiethnic primary schools in England with many low-income immigrants showed similar patterns with respect to devaluing the kinds of knowledge students possessed out of school and valuing school mathematics for the kinds of careers that have high status in society.

All of these studies have been qualitative and with fairly small numbers of students. However, the ethnographic approaches (data gathered from students, teachers, and community members as well as school observations and structured tasks) and the repeated patterns across sites provide a convincing picture that when it comes to mathematics class, students are implicitly taught to ignore their out-of-school experiences. Although these studies were conducted with primary school children, the findings they report seem plausible for older students as well.

The school walls could be more permeable, with teachers taking kids into the community to study how people use mathematics in their everyday lives or inviting community members into the school to talk about the kinds of things they do and how that relates to mathematics. Then students might gain a better sense of themselves as doing mathematics and, therefore, more interested in knowing how their practices relate to formal, abstract mathematics taught in school. Moreover, if students were encouraged to draw on their out-of-school experiences to offer multiple representations for the mathematics classroom, they might be more willing to see connections between their out-of-school mathematical practices and their in-school mathematical practices. Such approaches might also serve to position as mathematical experts students who were not previously seen as competent, based on school performance.

**SUMMARY**

Most of the studies on learning and using mathematics in everyday contexts focus on adults, not the adolescents with whom we are concerned. Not all studies indicate the ethnic or racial backgrounds of the learners or the locations of the studies. That makes it difficult to know how pertinent the findings from these studies are for Latino/a and black students. It also raises issues of how to apply this knowledge to school contexts.

Even so, findings from these studies raise important issues for teaching and learning mathematics with Latino/a and black students. For example, how might student learning differ if teachers asked students to keep a daily log of the mathematical practices outside of school in which they were engaged? How might this approach be combined with studying the histories of other cultures learning mathematics, thereby expanding students’ views of what mathematics is—and expanding the opportunities to see themselves using mathematics in their everyday lives?

How might these kinds of new school rituals help students build mathematical identities (e.g., see themselves as mathematics people)? Might students’ engagement change if teachers began with situations that were familiar, important, or in some way meaningful to their students and drew upon these funds of knowledge for launching mathematical explorations or modeling of phenomena with concrete objects?

Instead of relying upon the teacher or textbook publishers to design “real-world” problems, might mathematics learning look different if students were